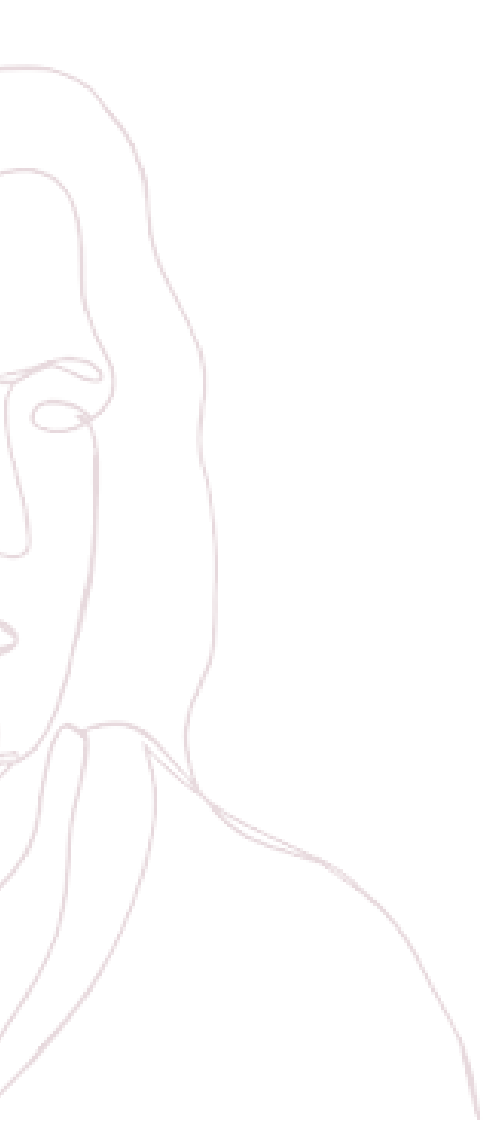




Discrete Probability Models

Statistics I

Disclaimer

These slides are a free translation and adaptation from the slide deck for Estatística I by Prof. Sandra Custódio and Prof. Teresa Ferreira from the Lisbon Accounting and Business School, Polytechnic University of Lisbon.

Types of random variables

- We studied a couple of ways in which we could categorize the random variables.
- One of these classifications was *discrete* and *continuous* random variables.
- We will start by focusing in the first family of *r.v.s*

Types of random variables

- According to the type of event we are measuring, we can do even better, and cluster together random variables that behave similarly.
- We will define *families* of **statistical distributions**. Knowing to which statistical distribution our random variable belongs, will endow us with a set of tools to deal with them adequately.

Bernoulli and Binomial Distributions

Bernoulli experiment (or trial)

The **Bernoulli experiment** consists in a random experiment with the following features:

- Only two events: A and A^c , for example *success* and *failure*.
 $\Omega = \{A, A^c\}$
- The success happens with probability p and failure with probability $q = 1 - p$:

$$P(A) = p \quad P(A^c) = q = 1 - p$$

Bernoulli Process

This process has even further these three characteristics:

- Only two possible outcomes in each draw
- $P(A)$ remains constant thorough the experiment
- Repeated trials are independent, *i.e.* what happened before does not affect what will happen in the future.

This is process has a binary outcome: Success (1) - Failure (0).

Bernoulli Distribution

The random variable X follows a Bernoulli distribution with parameter $p \in [0, 1]$, $X \sim \text{Ber}(1, p)$ if its probability function is given by:

$$P(X = x) = f_X(x) = \begin{cases} p^x(1 - p)^{1-x} & x \in \{0, 1\} \\ 0 & \text{otherwise} \end{cases}$$

It is trivial to check that $P(X = 1) = p$ and $P(X = 0) = 1 - p$.

Bernoulli Distribution

The first two moments of the distribution (mean and variance) are given by:

- $E[X] = p$
- $V[X] = p(1 - p)$

In summary, Bernoulli's tries to model what is the probability of success or failure in a single independent trial.

Binomial Distribution

Binomial Distribution

This distribution, is a **generalization** of Bernoulli's.

In this case, it might be better to start from the summary to stress the difference with Bernoulli's:

The Binomial distribution answers what is the probability that, after n independent fail-success trials, in any order, for example, you succeeded $m \leq n$ times.

Binomial Distribution

A random variable X follows a Binomial distribution, $X \sim \text{Bin}(n, p)$ if its probability function $P(X = x)$ is:

$$f_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

Note that if we change n or p we would get another binomial distribution, i.e. $B(n, p) \neq B(m, r)$ if either $n \neq m$ or $p \neq r$.

Bernoulli is a Binomial distribution with $n = 1$.

Side note about binomials

Binomial distribution

Computing this probability can become very laborious, even for small values for n . This can be solved thanks to:

1. Computers and advanced calculators
2. Tables for values for n from 1 to 20, and for p from 0.05 to 0.5. Note that if $p > 0.5$ you can just

$$X \sim \text{Bin}(x, n, p) \leftrightarrow \hat{X} \sim \text{Bin}(n - x, n, 1 - p)$$

Binomial distribution

The first two moments of the distribution are found as:

$$1. E[X] = \mu_X = \sum x \binom{n}{x} p^x (1-p)^{n-x} = np$$

$$2. V[X] = E[(X - \mu)^2] = \\ = \sum x^2 \binom{n}{x} p^x (1-p)^{n-x} - \mu^2 = np(1-p)$$

Additivity Theorem of the Binomial Distribution

Let k independent random variables X_i with $i = 1, 2, \dots, k$, where $X_i \sim \text{Bin}(x_i, n_i, p)$, then

$$\text{if } S_k = X_1 + X_2 + \dots + X_k = \sum_{i=1}^k X_i$$

$$S_k \sim \text{Bin} \left(s_k, n = \sum_{i=1}^k n_i, p \right)$$

Example

Using the [tables](#) for the binomial distribution, or the calculator, find the value of the following probabilities:

1. $P(X \leq 3)$ if $X \sim \text{Bin}(9, 0.4)$
2. $P(2 \leq X \leq 6)$ if $X \sim \text{Bin}(7, 0.3)$
3. If $X \sim \text{Bin}(10, p)$ and $P(X \leq 5) = 0.8338$, what is p ?

►  Answer

Example

The SNS (Portugal's National Health Service) has said that the probability of an individual carrying a specific gene, that triggers a certain disease is 0.1. In the city of Porto, 10 individuals were randomly selected for a study, while 8 individuals were randomly chosen in Lisbon for the same purpose. Assess the veracity of the following sentence:

The probability of, in Lisbon, 2 individuals carrying the gene, knowing that between Porto and Lisbon only 5 individuals were found to carry that gene, is 0.2941.

Example

Let $X \sim \text{Bin}(10, 0.1)$ (for Porto) and $Y \sim \text{Bin}(8, 0.1)$ (for Lisbon). Then $X + Y \sim \text{Bin}(18, 0.1)$.

$$\begin{aligned} P(Y = 2 | X + Y = 5) &= \frac{P(Y = 2, X + Y = 5)}{P(X + Y = 5)} \\ &= \frac{P(Y = 2, X = 3)}{P(X + Y = 5)} \\ &= \frac{P(Y = 2)P(X = 3)}{P(X + Y = 5)} \end{aligned}$$

Example

Using the [tables](#) we have $P(Y = 2) = 0.1488$, $P(X = 3) = 0.0574$, $P(X + Y = 5) = 0.0218$, and therefore $P(Y = 2|X + Y = 5) = 0.392$. ❌ False

? Bernoulli and Binomial · Question 1

If $X \sim \text{Bin}(n, p)$, then $E[X]$ and $V[X]$ are:

A. np and $np(1 - p)$

B. np and np

C. p and $p(1 - p)$

D. np^2 and np

 **A.** A Binomial is a sum of n independent Bernoulli trials, each with mean p and variance $p(1 - p)$.

? Bernoulli and Binomial · Question 2

A Binomial model requires:

A. draws without replacement

B. a continuous support

C. n independent trials with a constant p

D. a known λ

 **C.** Without replacement the probability changes from draw to draw, and that is the Hypergeometric, not the Binomial.



Bernoulli and Binomial · Question 3

A machine produces defective items with probability $p = 0.1$, independently.

In a sample of $n = 10$ items, compute $P(X = 0)$, $P(X \leq 1)$, $E[X]$ and $V[X]$.



Bernoulli and Binomial · Solution

$$P(X = 0) = \binom{10}{0} 0.1^0 0.9^{10} = 0.9^{10} \approx 0.3487$$

$$P(X = 1) = \binom{10}{1} 0.1 \times 0.9^9 \approx 0.3874, \text{ so } P(X \leq 1) \approx 0.7361$$

$$E[X] = np = 1 \cdot V[X] = np(1 - p) = 0.9$$

Hypergeometric Distribution

Hypergeometric Distribution

Suppose you have a population with size N , from where M elements have some characteristic, and therefore $N - M$ do not. From that sample, we draw randomly a sample of size n , without replacement, and we would like to know, from this sample, how many have the characteristic we mentioned.

Given we are not replacing the elements we put in our sample, the events here are not independent!

Hypergeometric Distribution

In this case the r.v. X would be how many elements from our sample have the referred characteristic. $X \sim \text{Hypergeometric}(N, M, n)$ if its probability distribution function $P(X = x)$ is:

$$f_X(x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$$

where $\max(0, M + n - N) \leq x \leq \min(M, n)$

Hypergeometric Distribution

The first two moments of the distribution are found as:

- $E[X] = \mu_X = n \frac{M}{N} = np$
- $V[X] = npq \frac{N-n}{N-1}$ note here that because we do not replace elements, we get a smaller variance.

Hypergeometric vs Binomial

- **Binomial**
 - Replacement
 - Same population every draw
 - Independent draws
- **Hypergeometric**
 - No replacement
 - Population changes every time
 - Dependent draws

Hypergeometric $\rightarrow$ Binomial

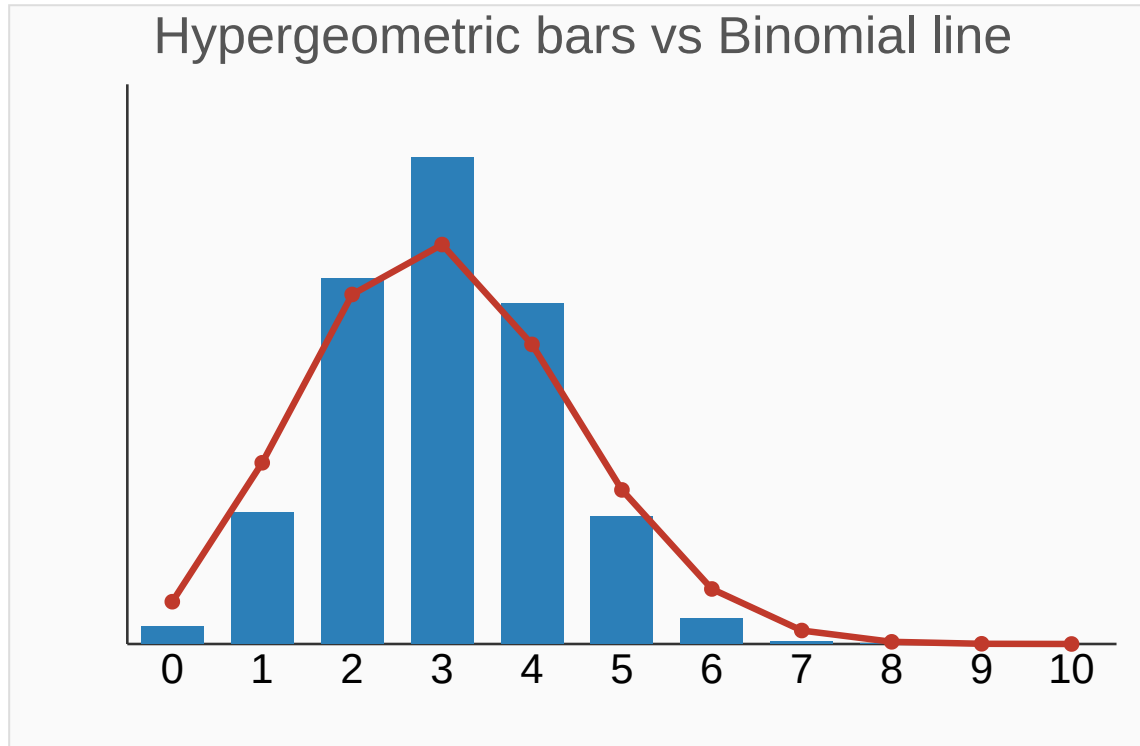
Let $X \sim \text{Hypergeometric}(N, M, n)$. If $N \gg n$ then the change in the population is very small, and then the draws approximate independent draws (population is very similar after a draw). As a rule of thumb if $\frac{n}{N} \leq 0.1$ or if the sample size is less than 10% of the population, we can approximate the Hypergeometric with the Binomial.

$$X \sim \text{Hypergeom}(N, M, n) \leftrightarrow X \approx \text{Bin} \left(n, p = \frac{M}{N} \right)$$

The smaller $\frac{n}{N}$ the better the approximation.

How big must the population be?

Bars are the exact $\text{Hypergeom}(N, M, n)$, drawing **without** replacement. The red outline is $\text{Bin}(n, p = M/N)$, drawing **with** replacement. The share M/N is held at 0.3 as N grows.



Population $N = 30$



Sample $n = 10$



$M = 9$, $p = M/N = 0.300$

$n/N = 0.333$

biggest gap = 0.0583

Rule of thumb: too small a population

needs $n/N \leq 0.1$

What the chart showed

With $N = 30$ and $n = 10$ you are removing a third of the population as you go, so each draw visibly changes the odds for the next one and the two distributions part company.

Push N past 100 with the same sample and the bars settle onto the red line: taking 10 out of 400 barely disturbs the population, so **draws without replacement start behaving like independent draws**.

That is all $\frac{n}{N} \leq 0.1$ is saying. It is not a statement about N being large, it is about the **sample being small relative to it**.

? Hypergeometric · Question 1

The Hypergeometric differs from the Binomial because the draws are:

- A. independent
 - B. infinite in number
 - C. continuous
 - D. without replacement, so p changes at each draw
-  **D.** Removing an item changes the composition of what is left, so the draws are not independent.

? Hypergeometric · Question 2

Approximating a Hypergeometric by a Binomial is safe when:

A. N is large

B. $\frac{n}{N} \leq 0.1$

C. $M = N$

D. $n > \frac{N}{2}$

 **B.** The rule of thumb is about the sample being **small relative to the population**, not about N being large on its own.



Hypergeometric · Question 3

A box holds $N = 20$ items, of which $M = 5$ are defective. You draw $n = 4$ without replacement.

Compute $P(X = 1)$, and say whether the Binomial approximation would be appropriate here.



Hypergeometric · Solution

$$P(X = 1) = \frac{\binom{5}{1} \binom{15}{3}}{\binom{20}{4}} = \frac{5 \times 455}{4845} = \frac{2275}{4845} \approx 0.4696$$

Here $\frac{n}{N} = \frac{4}{20} = 0.2 > 0.1$, so the Binomial approximation is **not** appropriate. ❌

Geometric Distribution

Geometric Distribution (Pascal's)

Consider a succession of Bernoulli trials, and let X the r.v. how many trials you need until observing the first success. This is a r.v. because there might be situations when you get success at your first trial, or second, etc. You cannot anticipate this.

$X \sim Geo(p)$ if its probability distribution function $P(X = x)$ is given by:

$$f_X(x) = p(1 - p)^{x-1}, \quad x \in \{1, 2, \dots\}, \quad p \in [0, 1]$$

Geometric Distribution (Pascal's)

The first two moments of the distribution are found as:

- $E[X] = \mu_X = \frac{1}{p}$
- $V[X] = \frac{1-p}{p^2}$
- $F_X(x) = \begin{cases} 0 & x < 1 \\ 1 - (1 - p)^k & k \leq x < k + 1, k \in \mathbb{N} \end{cases}$

Geometric Distribution (Pascal's)

There is a special feature of this distribution, it is said it “*lacks memory*” in the following sense. Let $s > t > 0$

$$\begin{aligned}
 P(X > s | X > t) &= \frac{P(X > s \wedge X > t)}{P(X > t)} = \frac{P(X > s)}{P(X > t)} \\
 &= \frac{1 - F(s)}{1 - F(t)} = \frac{(1 - p)^s}{(1 - p)^t} = (1 - p)^{s-t} \\
 P(X > s - t) &= 1 - P(X \leq s - t) = 1 - F(s - t) \\
 &= (1 - p)^{s-t}
 \end{aligned}$$

Geometric and Binomial

Distribution	r.v. X	Parameter
Binomial	# Success	# Trials
Geometric	# Trials	1 st Success

Example

A company specialized in sport footwear, imports a share of the material that is packed in boxes of 1000 units. To avoid counterfeiting, in each box 5 products are randomly selected, and the whole box is returned if one of these 5 products raises suspicions.

1. What is the probability that a box with 10 suspicious units is returned?

[Sol](#)

2. Check if this probability changes too much if you disregard that you are not replacing the items. [Sol](#)

3. What is the probability that we need to inspect 15 units, independently between them, until finding a suspicious item? [Sol](#)

4. How many units you inspect until finding the first suspicious unit? [Sol](#)

? Geometric · Question 1

$X \sim \text{Geo}(p)$ counts the number of trials up to and including the first success. Then $E[X]$ equals:

A. p

B. $\frac{1-p}{p^2}$

C. $\frac{1}{p}$

D. np

✓ C. If a success happens one time in five, you wait five trials on average. Note $\frac{1-p}{p^2}$ is the **variance**.

? Geometric · Question 2

The Geometric distribution is memoryless, which means that for $s > t$, $P(X > s | X > t)$ equals:

- A. $P(X > s)$
- B. $P(X > s + t)$
- C. $P(X > s - t)$
- D. 0

✓ **C.** We showed $P(X > s | X > t) = (1 - p)^{s-t}$, which is exactly $P(X > s - t)$. The failures already accumulated are forgotten.



Geometric · Question 3

Each call you make is answered with probability $p = 0.2$, independently. Let X be the number of calls until the first one is answered.

Compute $P(X = 3)$, $P(X > 4)$ and $E[X]$.



Geometric · Solution

$$P(X = 3) = (1 - p)^2 p = 0.8^2 \times 0.2 = 0.128$$

$$P(X > 4) = (1 - p)^4 = 0.8^4 = 0.4096$$

$$E[X] = \frac{1}{0.2} = 5 \text{ calls}$$

Poisson Distribution

Poisson Distribution

This distribution is associated with a process of counting, a Poisson process.

Examples:

- Count the number of patients arriving every day to the E.R. in a hospital.
- Count how many calls does a call-center receive in an hour.
- Count how many typos does a book have.

You can count something in a time frame, or a particular region.

Poisson process

The Poisson process has the following features:

1. It must be homogeneous in time/space. It only depends on the length that you consider to measure, not where, not when.
2. Events happening in disjoint regions/time slots must be independent.
3. The probability of having an event in the exact same space or at the exact same time is negligible (no simultaneous events).

Poisson Distribution

The r.v. X follows a Poisson distribution if $X \sim Poi(\lambda)$ if its probability distribution function $P(X = x)$ is given by:

$$f_X(x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!} & x = 0, 1, \dots \\ 0 & otherwise \end{cases}$$

with $\lambda > 0$, which represents the average number of events in a given time slot or region.

Poisson Distribution

The first two moments of the distribution are found as:

- $E[X] = \lambda$
- $V[X] = \lambda$

Additivity Theorem of the Poisson Distribution

Let k r.v.s X_i with $i = 1, 2, \dots, k$, independent, where $X_i \sim Poi(\lambda_i)$. Let

$$S_k = X_1 + \dots + X_k = \sum_{i=1}^k X_i$$

Then $S_k \sim Poi \left(\sum_{i=1}^k \lambda_i \right)$

Binomial $\rightarrow$ Poisson

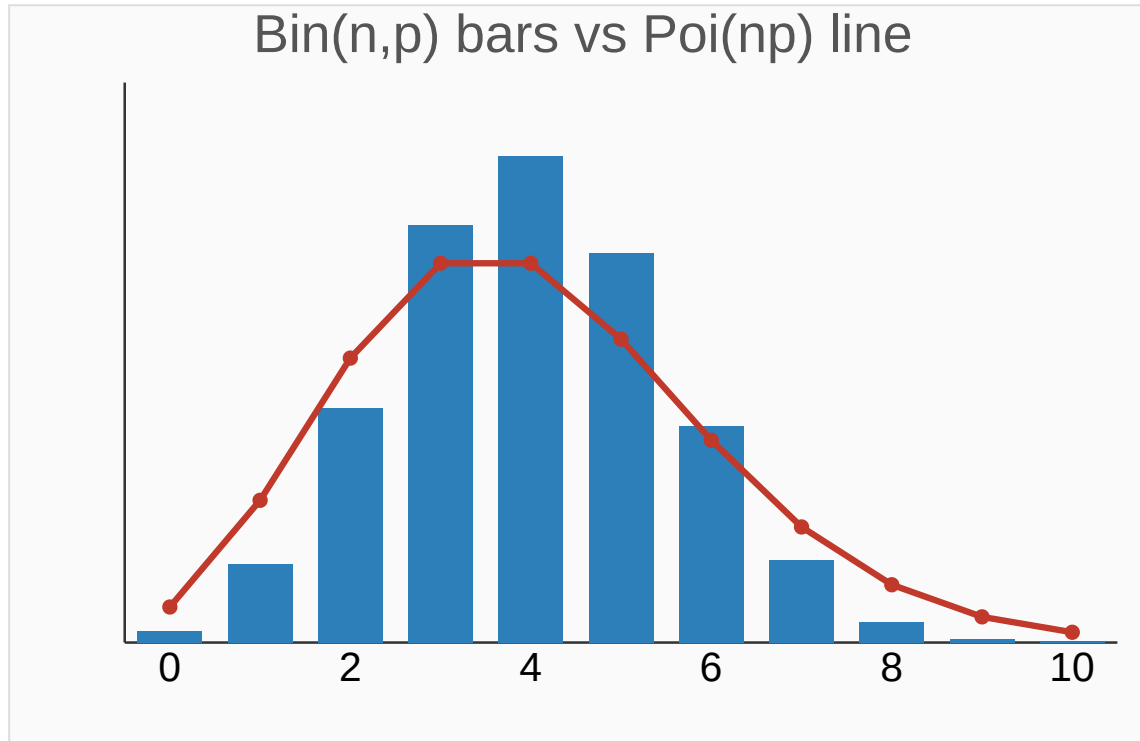
Let $X \sim \text{Bin}(n, p)$. If $n \gg 1 \gg p$, i.e. if this is a very rare event (very low success probability) in a very large sample, you can approximate this with a Poisson distribution, where $\lambda = np$.

$$X \sim \text{Bin}(x, n, p) \Leftrightarrow X \approx \text{Poi}(x, \lambda = np)$$

As a rule of thumb, do not approximate if $p \in [0.1, 0.9]$ or if $n \leq 20$.

When is the approximation safe?

Bars are the exact $\text{Bin}(n, p)$. The red outline is $\text{Poi}(\lambda = np)$. Slide n up and p down keeping λ near 4, and watch them merge.



$n = 10$



$p = 0.40$



$\lambda = np = 4.00$

biggest gap = 0.0555

Rule of thumb: do NOT approximate

needs $p < 0.1$ or $p > 0.9$, and $n > 20$

What the chart showed

At $n = 10$, $p = 0.4$ the red line misses the bars badly. Drag p down to 0.02 and n up to 200: same $\lambda = 4$, and the two become almost indistinguishable.

The approximation is not about λ being right, it is about **the event being rare in a large sample**. Keeping λ fixed while raising n and lowering p is exactly the limit the theorem describes.

Watch the “biggest gap” readout against the rule of thumb: it is what the $p \notin [0.1, 0.9]$ and $n > 20$ conditions are protecting you from.

Example

Find the value of the following probabilities, using the Poisson [table](#)

1. $P(X \leq 5)$ if $X \sim Poi(10)$
2. $P(4 \leq X \leq 8)$ if $X \sim Poi(5)$ [Sol](#)
3. $P(X = 7)$ if $X \sim Poi(10)$ [Sol](#)

►  Answer

Example

The number of patients arriving daily to the ICU in a hospital, follows a **Poisson** process with mean 4. The ICU has capacity of 6 patients, the others, are derived to the nearest hospital. Assess the validity of the following sentences:

1. The probability that, on a given day, there is no need to transfer any patient is 0.8893. **Sol**
2. The most likely number of patients arriving daily to the ICU is 6. **Sol**
3. The probability that, on a given day, arrive 5 patients, given that in the previous day only 2 patients arrived, is 0.1563. **Sol**
4. The probability that, in 5 days, at least 15 patients arrive to the ICU is 0.8435. **Sol**
5. To ensure that approx. 97% of the time (days) there are no transfers, it is necessary to increase the capacity in 4 more beds. **Sol**

? Poisson · Question 1

If $X \sim \text{Poisson}(\lambda)$, then:

A. $E[X] = \lambda, V[X] = \lambda^2$

B. $E[X] = \frac{1}{\lambda}$

C. $E[X] = \lambda, V[X] = 1$

D. $E[X] = V[X] = \lambda$

 **D.** The Poisson is the one distribution in this chapter whose mean and variance coincide.

? Poisson · Question 2

The Binomial is well approximated by a Poisson when:

A. n is large, p is small, and np stays moderate

B. n is small and p large

C. $n = p$

D. always

 **A.** Many trials, each very unlikely, with a stable expected count $\lambda = np$: rare events over a fixed interval.



Poisson · Question 3

A call centre receives on average 3 calls per minute, following a Poisson process.

Compute $P(X = 0)$, $P(X \leq 2)$ and $P(X > 2)$ for a given minute.



Poisson · Solution

With $\lambda = 3$ and $f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$:

$$P(X = 0) = e^{-3} \approx 0.0498 \cdot P(X = 1) = 3e^{-3} \approx 0.1494 \cdot P(X = 2) = \frac{9}{2}e^{-3} \approx 0.2240$$

$$P(X \leq 2) \approx 0.4232, \text{ so } P(X > 2) = 1 - P(X \leq 2) \approx 0.5768$$

Bibliography

- Murteira, B.; Silva Ribeiro, C.; Andrade e Silva, J. & Pimenta, C., Introdução à Estatística, Escolar Editora, McGraw-Hill, 2010
- Paulino, C. D. & Branco, J. A. (2005). Exercícios de Probabilidade e Estatística. Escolar Editora
- Pimenta, F., Andrade e Silva, J.; Silva Ribeiro, C. & Murteira, B., Introdução à Estatística, 3ª Edição, Escolar Editora, 2015

Appendix

Binomials

Back to Binomial Distribution

Binomial Table

[Back to exercise](#)

Distribution Function

$n = 1$

x	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 2$

x	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 3$

x	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
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Hypergeometric Example

Back to the exercise

First, note that $X \sim \text{Hypergeometric}(1000, 10, n)$

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) = 1 - P(X \leq 0) \\ &= 1 - P(X = 0) = 1 - \frac{\binom{10}{0} \binom{990}{5}}{\binom{1000}{5}} \\ &= 1 - 0.9509 \approx 0.0491 \end{aligned}$$

Hypergeometric Example

Back to the exercise

With reposition we could approximate with the Binomial (note $n/N = 0.005 < 0.1$), $X \approx \text{Bin}(n = 5, M/N = 0.01)$

$$P(X \geq 1) = 1 - P(X = 0) \approx 1 - 0.95099 \approx 0.04901$$

Both values are very close.

Hypergeometric Example

Back to the exercise

$$Y \sim \text{Geo}(p = 0.01),$$

$$\begin{aligned} P(Y = 15) &= p(1 - p)^{y-1} = 0.01(1 - 0.01)^{14} \\ &= 0.01 \times 0.8687458 \approx 0.0087 \end{aligned}$$

Hypergeometric Example

Back to the exercise

$Y \sim \text{Geo}(p = 0.01)$, then

$$E[Y] = \frac{1}{p} = \frac{1}{0.01} = 100$$

You need to inspect 100 units on average.

Poisson Table

[Back to exercise](#)

Distribution Function

$x \setminus \lambda$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$x \setminus \lambda$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0

Poisson Example

[Back to the exercise](#)

If X is the r.v. of the number of patients arriving every day at the ICU, $X \sim \text{Poi}(4)$. We are interested in $P(X \leq 6)$, looking at the table we get $F(6) = 0.8893$.  True.

Poisson Example

Back to the exercise

Note that from the table we can see that

$P(X = 3) = P(X = 4) = 0.1954$, and also $\lambda = 4$. Given that $\lambda = E[X]$, the most likely value must be λ or very close. In sum, the sentence is  false.

Poisson Example

Back to the exercise

We need to remember two important facts about the Poisson distribution. In two disjoint periods, the random variables are independent. So the number of patients one day, and the number of patients in the next days are independent. On the other hand, remember that the distribution must be the same for the same time-window (they are independently but identically distributed):

$$P(X_{day2} = 5 | X_{day1} = 2) = P(X_{day2} = 5) = 0.1563$$

The sentence is  true.

Poisson Example

Back to the exercise

Let $Y \sim Poi(20)$ (because of the additivity theorem).

$$\begin{aligned} P(Y \geq 15) &= 1 - P(Y < 15) \\ &= 1 - P(Y \leq 14) = 1 - F(14) \\ &= 1 - 0.1049 = 0.8951 \end{aligned}$$

The sentence is **✗** false.

Poisson Example

Back to the exercise

In this exercise, we need to find, given that $\lambda = 4$, the x that makes $F(x)$ greater or equal than 0.97. By inspection in the table, we see that it is $x = 8$, but we have know 6 beds, and therefore we would need 2 more beds to satisfy our requirement.

The sentence is  false.